Montessori Materialized Abstractions

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Just Who Was This Pythagoras, Anyway?

570 B.C. - 475 B.C

"In our everyday world we constantly see the basic shapes of squares, rectangles and circles. Each morning, as we step into the kitchen, we are surrounded by familiar shapes: the fridge is rectangular, the cereal bowls are circular, and there's a good chance that what we eat is square or rectangular. As we drive out into our neighborhood we see triangular yield signs, octagonal stop signs and numerous rectangles. These familiar, easily recognizable shapes provide us with a feeling of continuity and stability. Additionally. These shapes are basic and familiar.

Pythagoras, the Greek philosopher and mathematician who showed that the square of the hypotenuse is equal to the sum of the squares of the sides of a right triangle, is important to the world of design because Pythagorean numbers make shapes and ratios that feel familiar and comfortable to us." "All things are numbers".

Pythagoras was a Greek philosopher, born on the island of Samos. We can date his life back to between 580-520 BC. Pythagoras was a self motivating man. He received his education through his own thoughts and beliefs and therefore made his own assumptions. As a young man he did much traveling through Egypt, learning, among other things, mathematics. Not much more is known of his early years. He left Samos because of the tyrant who ruled there and went to southern Italy about 532 BC. to the Greek city of Croton, where he developed a philosophical and religious school which made many outstanding contributions to mathematics, astrology, and music. Here Pythagoras was a very active teacher of his time. He invented an 8-string lyre, taught astrology, music, mathematics, and more. Pythagoras and his students believed that everything was related to mathematics. They explained numbers and their properties by means of dots arranged in certain figures or patterns. In addition, Pythagoras believed that "Number rules the universe," giving numerical values to many objects and ideas. "All things are numbers".
He maintained that numbers are not only the symbols of reality, but the very substance of real things. He held, for example, that one is the point, two the line, three the surface, and four the solid. Seven he considered to be the fate that dominates human life, because infancy ceases at seven, maturity begins at fourteen, marriage takes place in the twenty-first year, and seventy years is the span of life usually allotted to man. Ten is the perfect number, because it is the sum of one, two, three, and four-the point, the line, the surface, and the solid.

Pythagoras viewed numbers as "personalities." For him numbers were masculine or feminine, perfect or incomplete, beautiful or ugly, in addition to even or odd. He believed that ten was the very best number, because it contained the first four integers (1+2+3+4=10). Having, naturally, observed that all numbers may be ranged in parallel columns under "odd" and "even", he was led to attempt a similar arrangement of the qualities of things. Under odd he placed light, straight, good, right, masculine; under even, dark, crooked, evil, left, feminine. These opposites, he contended, are found everywhere in nature, and the union of them constitutes the harmony of the real world.

Pythagoras is also famous for his study of sound and his theorem relating the lengths of the sides of a right triangle. Without his Pythagorean theorem, we couldn't have the computer graphics capabilities that we do now. Surveying and measuring land would be more difficult. Overall, Pythagoras was a philosopher who believed that numbers and geometry were the basis of all reality, and that the understanding of mathematics lifted the soul."

Pythagoras Multiplication Table

x	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

Montessori Multiplication Table

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	19	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

PYTHAGORAS' SQUARE

Age: 3 ½ and up

Materials:

- Pythagoras' square, a box, which contains 10, sets of wooden, colour codes squares (10) and rectangles (90) and a square frame with a molded edge, a mat. The colours of the rectangles and squares are corresponding to the coloured bead stair.
- A working mat.

Presentation:

1. Invite the child to the shelf.
2. Show the child where the box and the frame are kept in the environment.
3. Invite the child to carry the mat to an area on the floor and unroll the mat then put down the square frame + the box.
4. Invite the child to open the box and remove the small red square. Then point to the top left hand corner of the frame and invite the child to display it right in the corner.
5. Next invite the child to remove the green square and rectangles and to lay them on the frame. Then ask: "Do you see the square ? Could you put it here pointing to the lower right hand corner of the red square?
6. Then the child is invited to place the two small green rectangles on either side of the green square.
7. The teacher continues: "May you take out all the pink squares and rectangles?" making sure that only one coloured set is taken out of the box one at a time.
8. The teacher continues in the same manner: "Do you see the square ? Could you place it here" pointing to the spot where the child should place the pink square, which is at the bottom lower corner of the green square?
9. The child is then invited to place the two largest pink rectangles against the pink square, then the second largest rectangles against the previous ones, and so forth.
10. Invite the child to continue to build the Pythagoras' square in the same way with all the other coloured sets, till the end.
11. Make sure the child always starts with the square, then the largest rectangles against the square, then the next largest rectangles, and so forth, in decreasing order.
12. When completed, admire his/her work then invite the child how to return the pieces to the box. First, invite to place the square into the compartment first, and then overlie it with the largest rectangle + the smallest rectangle to form a square for each layer beginning by the golden set and finishing by the red set.
13. Have the child return the box and the frame back on the appropriate place on shelf.

Language: square, rectangle, largest, smallest, if needed

Direct Aim: To build a Pythagoras' square: discrimination of size, shape and colours

Indirect Aim:

Helping developing a logical mathematical mind
Eye-hand coordination
Build intellectual activity
Refine the senses and develop cognitive skills such as thinking, judging, associating and comparing
Develop powers of observation such as attention and concentration
Preparation for multiplication
Preparation for algebra $(1+2+3+4+5+6+7+8+9+10)^2 = (a+b+c+d+e+f+g+h+i+j)^2$

Point of Interest:
The frame, the diagonal position of squares, the same colours as in the short bead stair, the moment the Pythagoras' square is completed.

Control of Error:
Visual: the lines
Not handling the material properly; noise, stepping on the frame
Not placing the rectangles where they belong from the largest to the thinnest
Not stacking the materials back in the box with the square first and largest + smallest rectangles juxtaposing and constantly forming a square.

Activities before:
Presentation tray of the geometric cabinet
Geometric cabinet
Constructive triangles
Superimposition of shapes in practical life

Activities after:
Geometry
Multiplication board and tables
Bead cabinet

Variations:
1) With rectangles, we can create squares
2) Binomials and trinomials superimposed on the square of 10
3) Building a tower with all the squares of the Decanomial square
4) Geometric multiplication with any shape
5) Colouring the pattern of the Pythagoras' square

Extensions:
1) Building a tower with the squares of the bead cabinet
2) Superimposing the squares of the bead cabinet over the corresponding squares of the Decanomial square
3) Colouring on a squared paper the shapes superimposed
4) Finding the data corresponding to the factors of the Decanomial square e.g.:
   72= (4+25+36+7) or 72= (9+27+20+14+2) and so on.

The Decanomial square is the most valuable manipulative tool

PYTHAGORAS’ SQUARE (blank)

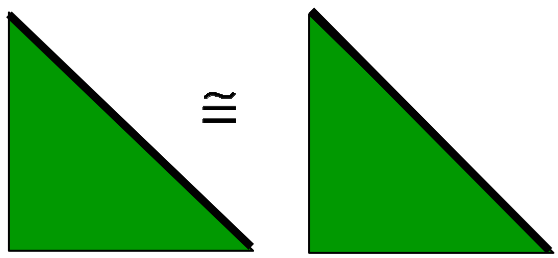
Constructive triangles

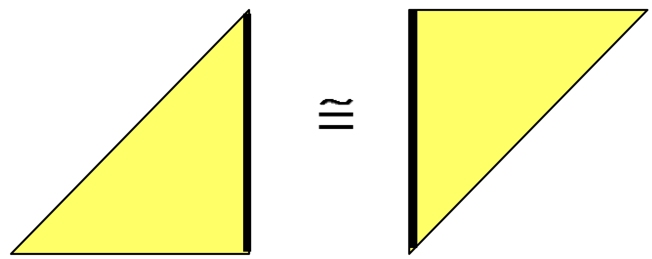
Concepts of congruence, similarity and equivalence

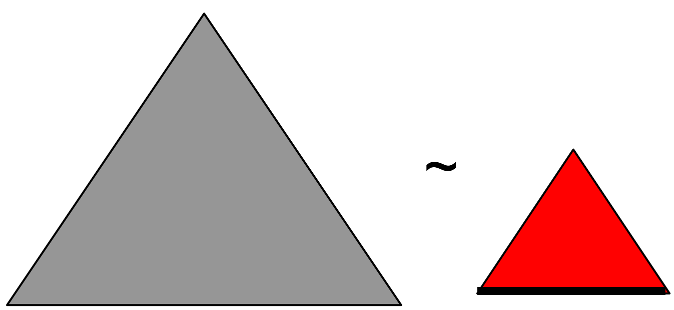
<u>Congruence</u>: When two geometric shapes have the same size (area) and shape - that is, if their corresponding angles and sides are equal. They are exactly the same shape and the same size. The symbol is ≅

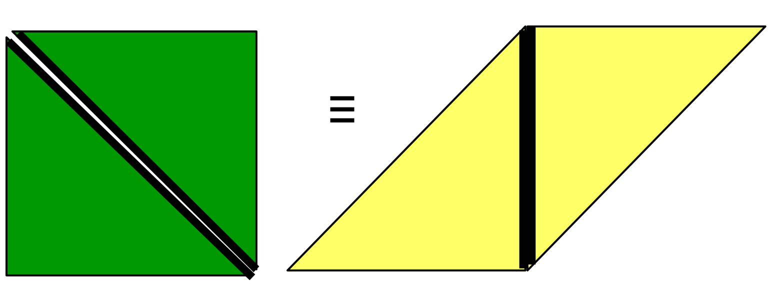
<u>Similarity</u>: When the sides of two geometric shapes are proportional that means they have proportions between their sides and angles. They are exactly the same shape, but they are NOT the same size (area). The symbol is: ~

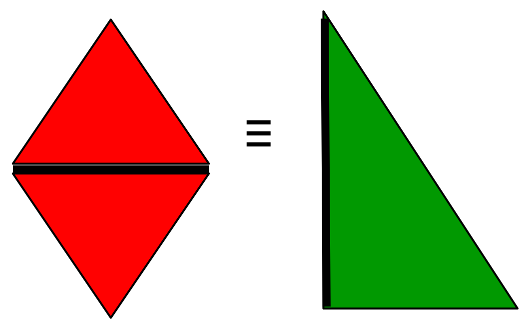
<u>Equivalence</u>: When two geometric shapes have the same size (area) but NOT the same shape. They are exactly the same size (area) and their shape is different. The symbol is: ≡





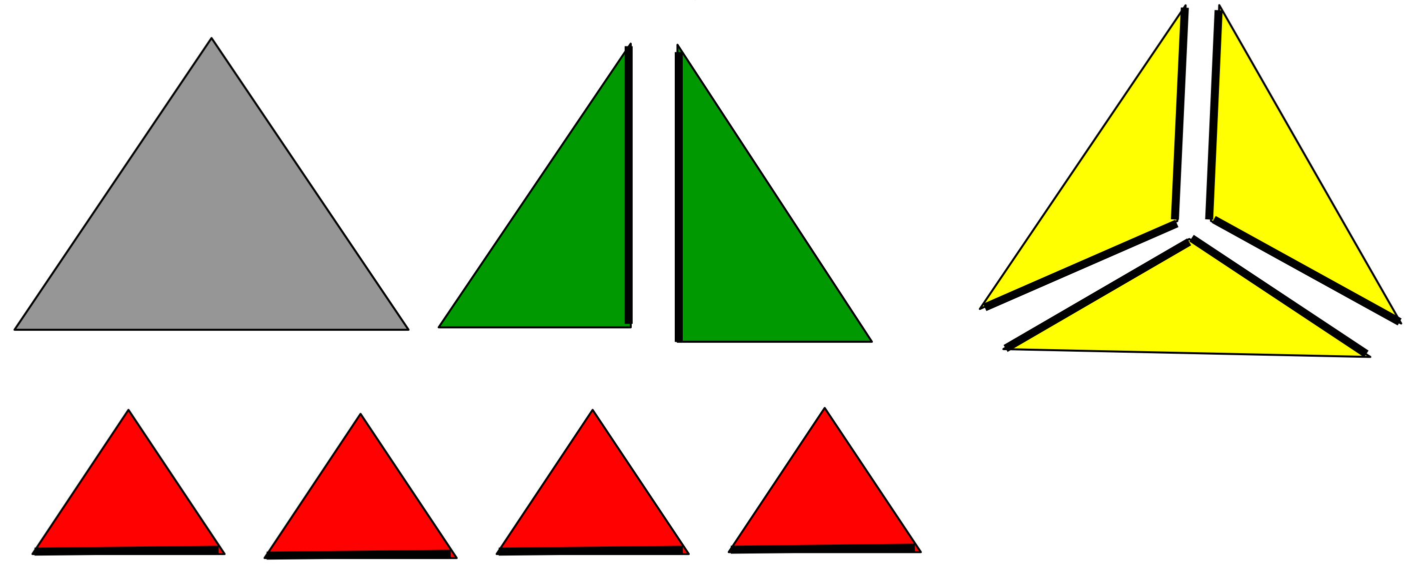


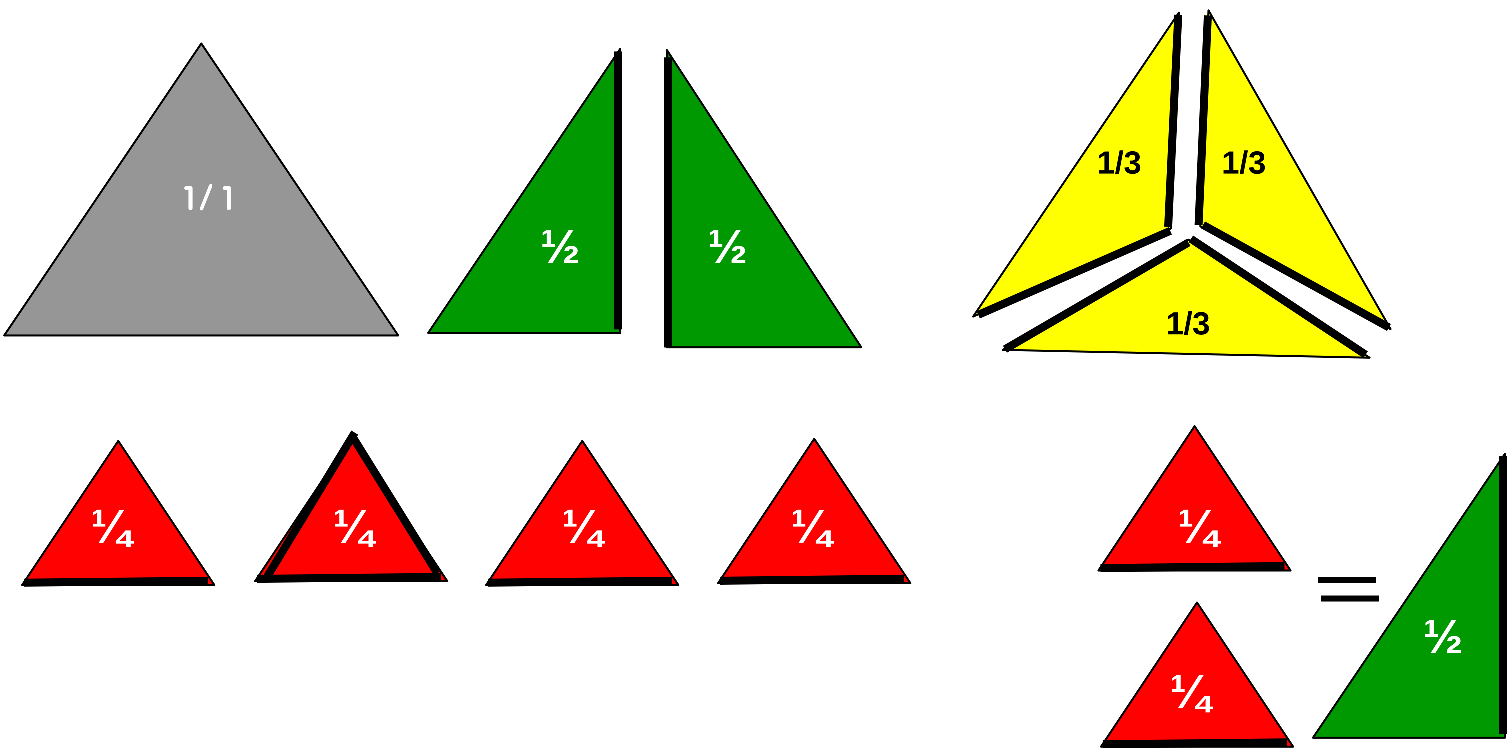


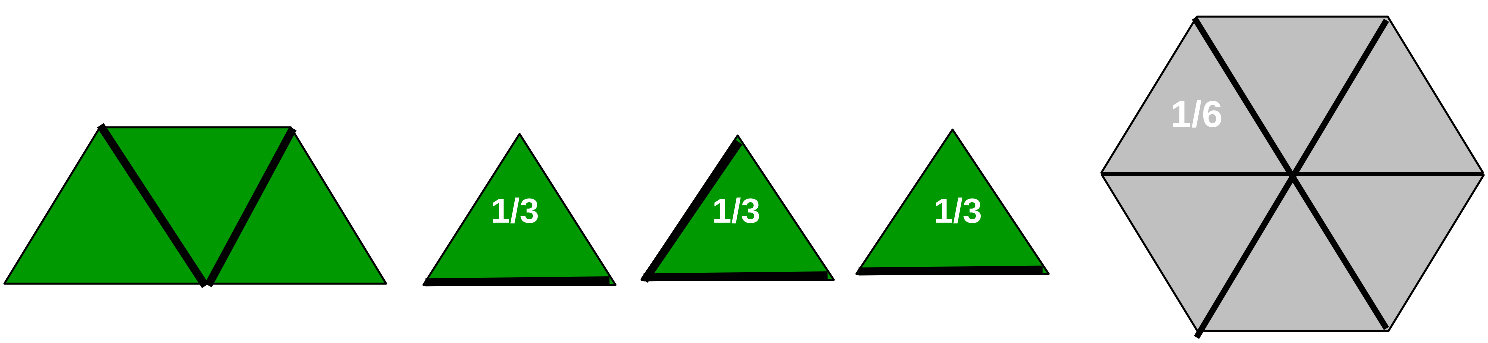


Constructive triangles

Fractions







Algebra with Pink Tower and Broad Stair

Pink Tower

Age: 4 + as long as the child has successfully worked with the blocks of the Pink tower and his movements are sufficiently coordinated.

Materials:
- Pink box containing 271 cubes in natural wood of 1 cm^3.
- Blocks of Pink Tower.
- Working mat.

Presentation:
1. Invite the child to the shelf.
2. Show the child where the box is kept.
3. Invite the child to carry the mat to an area on the floor and unroll the mat then put down the box.
4. Invite the child to bring the third cube (3 cm^3) of the Pink tower to the working mat.
5. Invite to open the box and to remove one of the small wooden cubes. Tell the child that all these small cubes are similar to the smallest one of the Pink Tower.
6. Next invite the child to build a similar cube (3 cm^3) with the small cubes and to compare and verify its dimensions by juxtaposing the pink cube of 3 cm^3.
7. Invite the child to build other cubes of other dimensions with the help of the small cubes contained in the pink box.

Another day…

1. Invite the child to bring the pink box of small cubes that he/she knows.
2. Invite the child to carry the mat to an area on the floor and unroll the mat then put down the box.
3. Invite the child to bring two cubes from the Pink tower, for example the third cube (3 cm^3) and the fourth (4 cm^3) to the working mat.
4. Invite to build with the small wooden cubes a cube similar to the fourth by juxtaposing layers of small wooden cubes over the third pink cube.
5. Next invite the child to compare and verify its cubic dimensions (4 cm^3) by juxtaposing the pink cube of 4 cm^3.
6. Let the child be "sensorially" conscious that to go from a cube to the next cube larger in dimensions, the volume of the previous cube increases when he/she builds around the previous cube three layers to cover three sides of the previous cube + three rows on top of each layer + one small cube to complete the cube similar to the 4 cm^3; the last small cube becomes the link between the three sides of the new cube.
7. You may invite the child to count the cubes used to go from a 3 cm^3 to a 4 cm^3. Answer: 37 small wooden cubes (37 + 27 = 64)
8. Invite the child to make the same experience with two other cubes of the Pink tower; with 271 small cubes in the pink box, the child can build the cube of 10 cm^3 using the previous one 9 cm^3 because of this algebraic formula: $3n^2 + 3n + 1$ in which n varies according to the edge of the pink cube.

Explanation:

n = the edge of the pink cube

3n = three layers of small cubes alongside the three sides of the pink cube

+ 1 = the last small cube completing the newborn cube

If we use the fourth cube (4 cm^3) to build the fifth one (5 cm^3), we will use 61 small wooden cubes which is the difference between the cube of 4 cm^3 and 5 cm^3. To verify, we know that the 5 cm^3 has 125 cm^3 and a 4 cm^3 has 64 cm^3, so the difference between 125 and 64 = 61.

Formula: $3n^2 + 3n + 1 = (3 \times 4^3) + (3 \times 4) = 1 = 48 + 12 + 1 = 61$

9. Have the child return the box and the cubes back at the appropriate place upon completion.

Language: cube, small cubes, Pink tower if needed

Direct Aim: To build cubes from cubes.

Indirect Aim:

Helping developing a logical mathematical mind
Eye-hand coordination
Build intellectual activity
Refine the senses and develop cognitive skills such as thinking, judging, associating and comparing
Develop powers of observation such as attention and concentration
Preparation for addition, multiplication
Preparation for algebra which is replacing numbers by letters

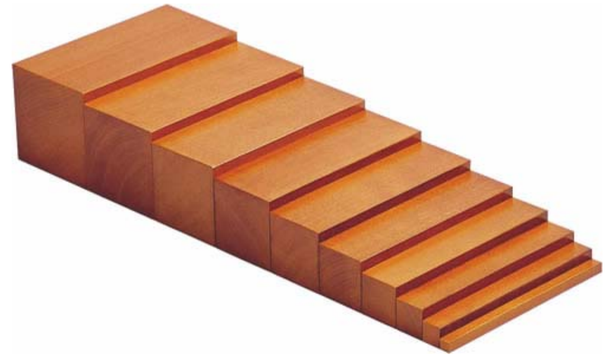
Point of Interest:
The small wooden cubes.
The moment the new cube is completed.

Control of Error:
Cubes of the Pink tower.
Visual

Activities before:
Building the Pink tower
Superimposition of shapes in practical life

Activities after:
Bead cabinet
Pythagorean square

Broad Stair



Age: 4 + as long as the child has successfully worked with the blocks of the Broad Stair.

Materials:
- Brown box containing 19 parallelepipeds in natural wood of 1 cm^2 by 20 cm.
- Blocks of Broad Stair.
- Working mat.

Presentation:
1) Invite the child to the shelf.
2) Show the child where the brown box is kept.
3) Invite the child to carry the mat to an area on the floor and unroll the mat then put down the box.
4) Invite the child to bring the third parallelepiped of the Broad stair to the working mat.
5) Invite to open the box and to remove one of the thin wooden parallelepiped. Tell the child that all these thin parallelepipeds are similar to the thinnest one of the Broad stair.
6) Next invite the child to build a similar parallelepiped (3 cm^2 x 20cm) with the thin parallelepipeds and to compare and verify its dimensions by juxtaposing the broad parallelepiped of 3 cm^2 x 20 cm.
7) Invite the child to build other parallelepipeds of other dimensions with the help of the thin parallelepipeds contained in the brown box.

Another day…

1) Invite the child to bring the brown box of thin parallelepipeds that he/she knows.
2) Invite the child to carry the mat to an area on the floor and unroll the mat then put down the box.
3) Invite the child to bring two blocks from the Broad stair, for example the fifth parallelepiped and the sixth to the working mat.
4) Invite to build with the thin wooden parallelepipeds a parallelepiped similar to the sixth one by juxtaposing layers of thin wooden parallelepipeds over the fifth brown parallelepiped.
5) Next invite the child to compare and verify its rectangular base prism dimensions (6 cm^2 x 20cm) by juxtaposing the sixth brown parallelepiped of the broad stair.
6) Let the child be "sensorially" conscious that to go from a parallelepiped to the next parallelepiped larger in dimensions, the volume of the previous parallelepiped increases when he/she builds around the previous parallelepiped two layers to cover two sides of the previous parallelepiped + one row to join the new sides completing the parallelepiped similar to the 6 cm^2 x 20 cm.
7) You may invite the child to count the parallelepipeds used to go from a 5 cm^2x 20 cm to a 6 cm^2 x 20cm. Answer: 11 thin wooden parallelepipeds
8) Invite the child to make the same experience with two other parallelepipeds of the Broad stair; with 19 thin parallelepipeds in the brown box, the child can build the parallelepiped of 10 cm^3 x 20cm using the previous one 9 cm^3 x 20 cm because of this algebraic formula: $2n + 1$ in which n varies according to the edge of the brown parallelepiped.

Explanation:

n = the edge of the brown parallelepiped

2n = two layers of thin wooden parallelepipeds alongside the two sides of the brown parallelepiped

+ 1 = the last thin parallelepiped connecting the two new sides of the parallelepiped

If we use the fifth brown parallelepiped (5 cm^2 x 20cm) to build the sixth one (6 cm^2 x 20cm), we will use 11 thin wooden parallelepipeds which is the difference between the fifth and the sixth. To verify, we know that the fifth uses 25 parallelepipeds and the sixth uses 36, so the difference is 11.

Formula: $2n + 1 = (2 \times 5) + 1 = 11 = 10 + 1 = 11$

10. Have the child return the box and the cubes back at the appropriate place upon completion.

Language: parallelepiped, thin, Broad stair if needed

Direct Aim: To build cubes from cubes.

Indirect Aim:

Helping developing a logical mathematical mind
Eye-hand coordination
Build intellectual activity
Refine the senses and develop cognitive skills such as thinking, judging, associating and comparing
Develop powers of observation such as attention and concentration
Preparation for addition, multiplication
Preparation for algebra which is replacing numbers by letters

Point of Interest:
The thin wooden parallelepipeds.
The moment the new parallelepiped is completed.

Control of Error:
Parallelepipeds of the Broad stair.
Visual

Activities before:
Building the Broad Stair
Superimposition of shapes in practical life
Combination Broad stair and Pink Tower

Activities after:
Geometric Cabinet
Pythagorean square

PROPERTY of ASSOCIATIVITY of NUMBERS

AGE: 6+

MATERIALS:

- A box containing many golden bead 10 rods and many coloured bead rods 1-9;
- Many brackets symbols, + and = signs;
- An container filled with equation slips for associativity;
- A mat.

PRESENTATION:

a) Invite the child to read one of the equation slip such as **3 + 5 + 7 + 8 + 5 =**

b) Invite the child to place the equation slip at the top of the mat and to display the beads according to the equation, the bracket symbols and the + and = signs as well.

c) Invite the child to display a second row of coloured beads below the first row but without the symbols and the signs.

d) Invite the child to think and figure what combination of numbers could make 10.

e) Invite the child to group the beads from the second row in order to make 10 , to put the bracket symbols between the combined beads and to bring vertically below a golden 10 rod.

f) Invite the child to group all the golden 10 rods and the remained color bead rod next to the + sign on the first row of the equation.

g) Encourage the child to write in his Math's notebook the equation and the answer as well.

h) Invite the child to explore many other equations slips.

CONTROL OF ERROR:

The answer written on the back of the slip.

DIRECT AIM:

1. To be skilled in the combinations of numbers making 10.

INDIRECT AIM

2. To encourage abstract counting.

6 + 9 + 3 + 4 + 1 =

9 + 5 + 1 + 5 + 7 =

4 + 3 + 5 + 7 + 6 =

4 + 2 + 9 + 8 + 6 =

9 + 5 + 2 + 8 + 1 =

3 + 1 + 7 + 9 + 5 =

9 + 4 + 1 + 3 + 6 =

8 + 9 + 2 + 4 + 1 =

5 + 8 + 5 + 8 + 2 =

4 + 3 + 9 + 7 + 6 =

9 + 2 + 2 + 8 + 1 =

4 + 1 + 8 + 9 + 6 =

1 + 5 + 2 + 8 + 9 =

1 + 2 + 9 + 8 + 6 =

2 + 5 + 2 + 8 + 5 =

4 + 2 + 7 + 8 + 3 =

8 + 5 + 2 + 8 + 2 =

4 + 3 + 9 + 7 + 6 =

9 + 6 + 2 + 4 + 1 =

PROPERTY of DISSOCIATIVITY of NUMBERS

AGE: 7+

MATERIALS:

- A box containing many golden bead 10 rods and many coloured bead rods 1-9;
- Many brackets symbols, + and = signs;
- An container filled with equation slips for dissociativity;
- A mat.

PRESENTATION:

a) Invite the child to read one of the equation slip such as

$3 + 9 + 9 + 8 + 5 =$

i) Invite the child to place the equation slip at the top of the mat and to display the beads according to the equation, the bracket symbols and the + and = signs as well.

j) Invite the child to display a second row of coloured beads below the first row but without the symbols and the signs.

k) Invite the child to think and figure which group of beads he/she could dissociate in order to get a combination of numbers that makes 10.

l) Invite the child to group the beads from the second row in order to make 10, to put the bracket symbols between the combined beads and to bring vertically below a golden 10 rod.

m) Invite the child to group all the golden 10 rods and the remained color bead rod next to the + sign on the first row of the equation.

n) Encourage the child to write in his Math's notebook the equation and the answer as well.

o) Invite the child to explore many other equations slips.

CONTROL OF ERROR:

The answer written on the back of the slip.

DIRECT AIM:

1. To be skilled in the combinations of numbers making 10.

INDIRECT AIM

2. To encourage abstract counting.

3 + 9 + 9 + 8 + 5 =

7 + 9 + 2 + 4 + 9 =

2 + 5 + 6 + 9 + 5 =

6 + 9 + 3 + 5 + 8 =

6 + 5 + 7 + 9 + 5 =

3 + 9 + 3 + 8 + 1 =

2 + 9 + 7 + 9 + 7 =

1 + 8 + 5 + 7 + 9 =

4 + 5 + 7 + 8 + 7 =

5 + 9 + 3 + 6 + 9 =

9 + 5 + 7 + 6 + 3 =

6 + 3 + 3 + 5 + 8 =

4 + 5 + 7 + 8 + 7 =

5 + 9 + 3 + 6 + 9 =

4 + 2 + 7 + 9 + 7 =

9 + 5 + 3 + 6 + 8 =

3 + 5 + 7 + 8 + 7 =

3 + 9 + 8 + 6 + 5 =

TRINOMIAL CUBE

Age: 3 ½ and up

Materials:

- The smallest and the largest cubes of the Pink tower;
    - Box containing 1 red cube, 1 blue cube, 1 yellow cube;
    - 3 thick and 3 thin red and black prisms;
    - 3 thick and 3 thin blue and black prisms;
    - 3 tall and 3 short yellow and black prisms,;
    - six all black prisms;
    - a felt with a glued pattern simulating a river.

Presentation (from the Maria Montessori's story in The Discovery of the Child, Ballantine Books p. 278)

1. Invite the child and show him where the trinomial cube is kept. Invite to work at a table or on a mat.

2. Have the child carry the box to the table or to the mat and set it so the hinges of the box are in front of you and invite the child to unroll the felt.

4. Invite to unroll the special felt, take the lid off, place it on the table so he can see the pattern.

3. Begin by telling the child: "*Do you remember the little castle we built before with two kings? Now this is the story of three kings. Would you like to hear the story?"*

4. Tell the story and invite to perform as the story goes on and on..

5. Invite to take out all the blocks one by one by using the 3-finger grip and placing them at random on the top part of the felt.

6. Invite to isolate the red cube, the blue cube and the yellow cube. *"You remember how each king could organize his entourage, do you?"* The child understands the term " entourage" because he has already built the binomial cube.

7. Invite the child to place by sizes all the red prisms under the red cube, all the blue prisms under the blue cube and all the yellow prisms under the yellow cube. There should be three rows, side by side.

8. Then you say: *"Each king has two bodyguards dressed in black!*" showing the black prisms. Invite to place each prism at either side of the red, blue and yellow cubes.

9. As the child has already built the binomial cube, so he has a good idea how to build this trinomial cube. The directress/director is there to present the "story of the three Kings and the King of Peace". When the story is over and the child is done, invite to marvel his work.

10. If he wants to do it again, invite to disassemble it and place the cubes and prisms at random on the felt and repeat the building of the castle according to the "story".

11. When finished, he lifts the two sides up from the box and places the lid back on the box (or he places the box-lid on the blocks forming the trinomial) and rolls the special felt.

12. Invite to return the box back on the shelf.

Language: The story of the three kings (Ref.: The Discovery of the Child, Ballantine Books p. 278)

Direct Aim:
- Visual discrimination of size and colour
- Sensorial construction of a cube

Indirect Aim:
- Development of a logical-mathematical mind
- Preparation for algebra
- To form judgment, to reason, to initiate comparisons and to decide
- To build a foundation for intellectual activity
- To refine his power of observation and to educate the eye to distinguish differences in dimension
- Develop sense-perception: child forms a visual image of the arrangement of the blocks and can thus remembers their quantity and order
- To develop imagination

Point of Interest:
- The "Montessori story" of the three Kings, their respective entourage and the King of Peace and his personal ambassador.

- The pattern on the lid or bottom of the box

- The colours and shapes of the cubes and prisms

Control of Error:
- If the cube is not formed and the "pattern" is not seen from any side of the cube: when completed, the pattern of the trinomial cube repeats itself 18 times: 6 times in each of the external faces, and 2 times in each of the internal sections (the cube can be separated into two parts, twice in each dimension, or into two parts, one each dimension).

- Noise/banging

Activities before: Binomial cube, sorting by colour, cylinder blocks

Activities after: Geometric cabinet, Decanomial square, geometric solids

Note: The 27 blocks per se represent the elements of this algebraic formula:

$(a+b+c)^3 = (a+b+c) \times (a+b+c) \times (a+b+c)$, that is:

$$a^3 + 3a^2b + 3ab^2 + b^3 + 3a^2c + 6abc + 3b^2c + 3c^2a + 3c^2b + c^3$$

The child is of course not introduced to the formula. He is only interested in the colour and sizes of the blocks (cubes and prisms) and how he can build a big cube.

Story of the Three Kings

Once upon a time, there were four kings living separately with their royal suite in four respective kingdoms: three on one side of a river and one on the other side. The three kings living on the North side of the river in three different castles were the Red King; the Blue King and the Yellow King. Each of the three kings had an entourage of two bodyguards dressed in black and six attendants. On the South side of the river lived the King of Peace in a pink tower with his own suite.

One day, the smallest attendant of the King of Peace was named ambassador and sent to make a courtesy visit to all three kingdoms on the North side of the river. During his visit, the ambassador perceived that each king and his entourage were unhappy. Apparently, the red castle was too hot; the blue castle was too cold and the yellow castle was too small. The ambassador came back to the pink tower and informed the King of Peace of the state of affairs of the three kingdoms.

In order to resolve the situation, the King of Peace in his kindness, prepared a plan to be presented to the three kings. If the three kings were to accept to live with their entourage in the same castle, but on a separate floor, in security, there will be harmony and happiness in each royal kingdom. The King of Peace sent his ambassador back to cross the river and present this plan to the three kings. At the approach of the ambassador, the Red King commanded all his attendants and bodyguards to be in position to receive the ambassador. His bodyguards and attendants realigned themselves.

The ambassador presented the King's plan to come to live in a brand new three stories castle. The Red King's immediate answer was: *"No!"* pretending he might be annoyed by the proximity of the two other kings: the Blue King and the Yellow King. He then said to the ambassador: "I prefer to stay in a castle which is too hot!" The ambassador then went to the Blue King to make the King of Peace's proposition. The Blue King commanded all his attendants and bodyguards to be in position to receive the ambassador. Unfortunately the king answered: *"No!* I prefer to live in a castle which is too cold than to share a castle with two royal neighbours!" Finally, the ambassador went to see the Yellow King. In order to receive the ambassador with dignity, the Yellow King commanded all his attendants and bodyguards to be in position to receive the King of Peace's ambassador. On behalf of his king, the ambassador proposed the Yellow King to lead his suite to dwell in a brand new castle and promised him that he would not be annoyed by any neighbour. Unluckily, the ambassador received the same negative answer.

Discouraged, the ambassador went back to inform the King of Peace of this unanimous refusal. The King of Peace met all his advisers in the tall pink tower and they arrived to a clever solution: each kingdom would live independently on a separate floor conditionally that each other kingdom has a spy who will reside permanently on each other floor of the brand new castle.

The King of Peace sent back his ambassador to the three kings to present this innovative proposal. The Red King received the ambassador and agreed to the deal. Protected by both bodyguards and four of his attendants, he led his royal suite leaving behind two of his own attendants chosen to become spies. The Red King chose to reside in the farthest corner of the first floor of the new castle. As the deal was that each king has to accept to be located close to, but not next to two special spies, two spies joined his suite: one from the blue kingdom and one from the yellow kingdom. When everybody was installed, the ambassador then went to visit the Blue King in his cold castle and invited him to move in high security with his suite on the second floor of the new castle, but not exactly over the Red King, but in the centre of the room. The Blue King accepted the compromise of being convivial with two unalike spies, issued from each royal red or yellow kingdom. The Blue King established his suite on the second floor after settling in the centre of the room. Then, the ambassador went to see the Yellow King and did the same claim: *"Would you accept to move and stay in tight security on the 3rd floor? You will not be staying over the Blue King nor the Red King."* The Yellow King who was living in a tight castle accepted all the conditions of the King of Peace's arrangement. His entire royal suite accompanied him with the two remaining spies after he has chosen for himself a far corner of the third floor of the new castle.

From then on, the Red King, the Blue King and the Yellow King lived separately in security, secluded on each floor, protected by their own bodyguards and attendants but sharing convivially a spy from each other royal kingdom. The King of Peace's plan brought harmony and happiness for ever. Therefore, the King of Peace's ambassador returned to his kingdom and positioned himself as an observer, till the next mission, at the very top of the famous Pink Tower.

The End